The meaning of justified subjectivism and its role in the reconciliation of recent disagreements over forensic probabilism

The meaning of justified subjectivism and its role in the reconciliation of recent disagreements over forensic probabilism

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Abstract

In this paper we reply to recent comments in this Special Issue according to which subjective probability is not considered to be a concept fit for use in forensic evaluation and expert reporting. We identify the source of these criticisms to lie in a misunderstanding of subjective probability as *unconstrained* subjective probability; a lack of constraint that neither corresponds to the way in which we referred to subjective probability in our previous contributions, nor to the way in which probability assignment is understood by current evaluative guidelines (e.g., of ENFSI). Specifically, we explain that we understand subjective probability as a *justified* assertion, i.e. a conditional assessment based on task-relevant data and information, that may be thought of as a *constrained* subjective probability. This leads us to emphasise again the general conclusion that there is no gap between justified (or, reasonable) subjective probability and other concepts of probability in terms of its ability to provide assessments that are soundly based on whatever relevant information available. We also note that the challenges an expert faces in reporting probabilities apply equally to all interpretations of probability, not only to subjective probability.

Keywords: Subjective probability, Probability assignment, Justification, Evaluative precepts

1. Introduction

What started as a discussion in [35] of what, at first sight, might look like a simple question – is there an interval around a probability, and hence around a likelihood ratio (Bayes factor)? – grew into a broader exchange in this *Science & Justice* Virtual Special Issue, involving forensic scientists, statisticians, lawyers, psychologists and cognitive scientists. This exchange was accompanied by a widening and interesting enrichment of the discussion, though it also gave rise to shifts of focus and revealed some incompatible positions. This is not surprising, given the diversity of backgrounds from which the discussants write, the differences in the nature of their respective disciplines, and the divergent views that occur even within some of these disciplines.

It is of interest to consider the outcome of this discussion. In this, our latest, contribution to this exchange of views, we seek to tease out some overall concepts and summary comments that we perceive from reading all the contributions published so far in this collection. We will have less to say about our original discussion topic – the understanding of probability and the likelihood ratio as a single number – because our position follows mainly from the full subjective Bayesian theory, and it is also acknowledged now in recent contributions to this article collection [e.g., 29]. We will devote more space here, in Section 2, to comments concerning personal, or subjective, probability that other discussants have made in this series of papers. We analyse and discuss eight points in turn.

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- Why it is important neither to confound nor to oppose the theory (of probability) with its application or perception (Section 2.1);
- Why it is important to properly define the scope of expert probability assertions (Section 2.2);
- Why the question of the foundations of assigned probabilities is important to all interpretations of probability, not only to subjective probability (Section 2.3);
- Why degrees of belief are scientific and can be empirically investigated (Section 2.4);
- Why and how any probability statement can always be reconstructed as a subjective probability (Section 2.5);
- Why the subjective interpretation of probability is a justifiable premise (Section 2.6);
- Why existing precepts for evidential assessment are sufficient to engage the entire field of forensic science (Section 2.7).
- Why it is important to understand the personalistic position as part of the concept of probabilistic modelling (Section 2.8).

Conclusions are presented in Section 3.

2. Selected discussion points

2.1. Why it is important neither to confound nor to oppose the theory (of probability) with its application or perception

The main theme in this article collection is that of the construction of intervals for probabilities and likelihood ratios. Consideration of this theme requires one to ask what probability theory says on this topic. Briefly, the answer to this is that [e.g., 5, 11], in their most fundamental form (axiomatic definition), the three rules of probability, say *nothing* about

- intervals,
- precision,
- application of the theory,
- assignation of probabilities.

As well as consideration, in a strict sense, of statements of the rules of probability, we have extended our discussion to various conceptually feasible devices for the measurement of probability, such as urn models (to elicit indifference points) or scoring rules [24]. In combination, these provide us with several independent routes that all lead to the same conclusion that probability is a single number [9, 12, 35].

This conclusion has several implications. Of course, one is free to say that one does not find these definitional properties of probability intuitive. However, it is important to keep in mind then that such a finding is immaterial to the logic of the theory. In other words, finding that one's intuitive feeling or thinking about probability does not conform to the properties of the theory cannot be claimed to be a problem of the theory [10]. Similarly, we do not consider arithmetic faulty simply because we may find it hard to accept that, for example, $2 + 2 = 4$.[1] The point here is that probability (theory) does not claim to provide a *description* of our mindset. The theory is merely providing a framework *to help* us organise our thinking in a coherent way. The essential challenge thus is to find ways for proceeding practically given the understanding of what the theory says and does not say. Nordgaard has clearly recognised this difficulty: "She might be convinced that it can only be one probability (...) but it is harder for her to be convinced that she actually has found it." [30, at p. 20] We conclude from the above that in order to avoid a misguided debate – i.e., the criticism of the probability for something that it does not purport to do – we should neither confound nor oppose the theory (of probability) with its application or how it is perceived. The two topics are distinct, though related, and the challenge for the forensic practitioner is find ways to apply the theory meaningfully.

2.2. Why it is important to properly define the scope of expert probability assertions

The previous discussion point highlights the importance of properly defining the scope of an expert's probability assertion. Also relevant for this is the consideration that probability, at least in its interpretation as a measure of belief,

[1] On this point, see also Lindley in his foreword for [13].

is essentially a person-related concept: probability serves as a measure of uncertainty from an individual point of view – but also taking account, where available, of domain-wide general knowledge accepted across different experts. The assertions of a scientist about probability thus deal with the scientific observations within the scientist's area of competence, and set aside other intricacies that the legal process may entail. We emphasise this point because of a cautionary remark formulated by Martire et al. [28] on the expression "encapsulating all uncertainty", introduced earlier in this Special Issue by Berger and Slooten [5]. Martire et al. [28] write that they "(...) question the extent to which a likelihood ratio based on personal probabilities can realistically be expected to 'encapsulate all uncertainty'." In defence of Berger and Slooten it should be emphasized that the scope of their expression 'all uncertainty' relates to the individual expert's point of view, whereas Martire et al. [28] appear to have in mind a broader perspective. In fact, in [28] they write: "(...) we believe there is additional uncertainty to be considered." The latter comment reminds one of the fact that a scientist's report is not the end of the matter. Note though that the comment should not be read as a suggestion that individual probabilities asserted by scientists for the scientific issues on which they are called to opine are deficient.

2.3. Why the question of the foundations of assigned probabilities is important to all interpretations of probability, not only subjective probability

The delicate intersection between, on the one hand, understanding of the properties of probability theory (Section 2.1) and, on the other hand, coping with this understandings in practical proceedings is well brought to our attention by Martire et al. [28]. They clearly acknowledge the reality that "(...) beliefs expressed as coherent probabilities are neither 'right' or 'wrong' (...)" [28]. But, they then go on to note that scientists "(...) sincerely stating subjective beliefs (...) raise problems for courts (...)" [28] in the sense that they need to know the exact worth of the probabilities they report. The question of trust that should be placed in a probabilistic statement proffered by a scientist in a particular instance is an essential element of a competent review of expert evidence by both prosecution and defence. However, we disagree on two aspects.

The first aspect regards the suggestion that the question of warrants[2] for probability assertions only affects subjective probabilities. However, this is a legitimate issue for enquiry regardless of the way – subjective or otherwise – in which scientists understand their probability statements. The real issue under discussion is not subjective probability, but probability assignments in general.

The second aspect regards the suggestion – as we understand it from [28], but also [29], both quoting [32] – that subjective probability is equivalent to guessing – i.e., *unconstrained* subjectivism. We have repeatedly argued why this does not reflect our position. Below, we thus restate and summarise a series of points that we have hoped to have clarified in our earlier contributions in this collection (see, in particular, Section 2 in [12]):

- First and foremost, we entirely concur with the critics of subjective probability that it is undesirable when scientists use the liberal concept[3] of probability in an arbitrary and speculative way, to make fanciful probability statements devoid of any meaningful justification. For example, blank statements such as 'in my opinion the probability of observing this particular configuration of minutiae, given that the fingermark has been left by an unknown finger, is so low that it can be considered a practical impossibility' is clearly going above and beyond what is scientifically justifiable. It is an assertion of a very small probability, rounded off to zero. In our previous contribution [12] we have thus noted that where "(...) probabilities are given in this way, this is a cause of concern, and [that] we would agree then with Morrison and Enzinger (...) that such probabilities should not be held acceptable". But, we have also noted that this should not lead us to reject subjective probability altogether because this would deprive us from the "(...) informed and defensible usage of belief type probabilities (...)" [12, p. 82]. We will have more to say about this, as it relates to the notion of 'justified (or, constrained) subjectivism' mentioned in title of this article.

[2] This term, mentioned in [28], is widely used in the context of argument analysis where it is understood as the provision of a justification as to why a particular conclusion of an argument is supported.

[3] We use the expression 'liberal concept' here because, as mentioned in Section 2.1, ~~the~~ probability theory does not prescribe us how to assign a particular probability, beyond defining the range of values that a probability may take.

- The requirement for expert probabilities to be well supported is mentioned explicitly as a requirement in current guidelines. For example, in its Guidance Note 3, the ENFSI Guideline for Evaluative Reporting in Forensic Science specifies [*emphasis added* by the authors]:

  > "The *basis for these assignments* shall be documented on the case file. Relevant and appropriate published data will be used wherever possible. If appropriate published data are not available then data from unpublished sources may be used. Regardless of the existence of sources (published or not) of numerical data, personal data such as experience in similar cases and peer consultations may be used, provided that the forensic practitioner can *justify the use of such data*." [16, at p. 15]

  > "Such personal probability assignment is not arbitrary or speculative, but is based on a body of knowledge that should be available for auditing and disclosure. The forensic practitioner should not mislead the recipient of expert information as to the basis of the personal assignment, and the extent to which the *assignment is supported* by scientific research." [16, at p. 16]

  The guideline also clearly states that if scientists cannot meet these requirements, then they can provide no help in assessing probative value:

  > "Note that if a likelihood ratio cannot be assigned by the forensic practitioner (due to a lack of knowledge for example), then no appropriate evaluative assessment of the findings can be made." [16, at p. 15]

- By subjective (or, personal) probability – the representation of an individual's state of knowledge (or belief state) about an uncertain event – we mean probabilities that have been determined with the use of transparent and coherent belief updating procedures using any relevant data there may be [7]. This is known in (philosophical) literature as the constraint of *conditioning*: the probabilities are conditioned on relevant information (data). Hence, our subjectivism is not *unconstrained*, and liable to the criticism of guesswork, but what may be called justified subjectivism. Our subjective probabilities are conditional probabilities and *at least* as data-informed as any other conceptualization of probability. A valuable feature of conditional probabilities is that they come with a clear explanation of how they have been obtained. Thus they conform to the ENFSI requirements mentioned above. In addition, criticism of subjective probability that reduces probability to its potential to permit arbitrary probability statements overlooks the potential of this concept to encourage scientists to adopt a disciplined approach and assume responsibility in the process of evidence evaluation.
- Interpreting probabilities as personal beliefs – i.e., subjective probabilities – does *not* mean to refute frequency or symmetry arguments central to other interpretations of probability. As noted by De Finetti:

  > "Those interpretations of the notion of probability (...) which are based on symmetry (the classical conception; equally likely cases), or on frequency (the statistical conception; repeated trials of a phenomenon), provide criteria which are also accepted and applied by subjectivists (...). It is not a question of rejecting them, or of doing without them; the difference lies in showing explicitly how they always need to be integrated into a subjective judgment, and how they turn out to be (more or less directly) applicable in particular situations." [14, at p. 334]

  Besides, there are also those cases – and there are many of them in forensic practice (see also Section 2.7 for an example) – in which relative frequency considerations or exchangeable events cannot be considered. Subjective probability can cope with such cases, whereas other interpretations of probability, namely the frequentist view, encounter applicability problems.

In summary, thus, we insist on subjective probability being a much more elaborate and richer concept than the alleged arbitrariness to which its critics reduce it. In particular, *justified subjectivism*, as we understand it, is based on probability that is as much informed by data – and is required to be so by current guidelines in the profession – as any other interpretation of probability. Further, the issue of questioning the warrants for particular probability assignments, as mentioned by Martire et al. [28], is a general one, affecting *any* expert probability assignment.

2.4. Why degrees of belief are scientific and can be empirically investigated

It is not uncommon to encounter views according to which degrees of belief are not scientific. Again, such views may be rooted in the idea that any superficially chosen probability value could be given (i.e., guessing). We have argued

in Section 2.3, and previously in [12], why this is not the case. Notwithstanding that argument, recent positions in this Special Issue reiterate the view that the subjective interpretation of probability is incompatible with a logical and scientific approach. Morrison, for example, writes that "The position that only a subjectivist concept of probability is legitimate" is "(...) counterproductive for the goal of having forensic practitioners implement, and courts not only accept but demand, logically correct and scientifically valid evaluation of forensic evidence" [29].[4]

There are two challenges in this objection, logic and scientific character (scientism). The former is readily dealt with: subjective probability complies with the rules of probability (i.e., the numbers that probabilities may take and how probabilities are to be combined), and hence is coherent. The latter requirement, scientism, requires more consideration, though we can only mention the topic briefly here. One way to comply with the requirement of scientism is to demonstrate that an individual's personal beliefs can actually be measured, for measurement is an essential element of science. Literature on belief measurement goes back several decades already to writers such as Savage [33] who described examples of experiments that allow one to learn about the subjective probabilities held by individuals. The basic idea of the proposed techniques for measuring degrees of belief involves a consideration of the bets that a person is willing to accept. We are not suggesting here that forensic practice should become a betting exercise, a point that we have also made in our previous discussion on scoring rules for probability assessment in a decision-theoretic perspective [12]. We are only making the conceptual point that – in principle – personal beliefs are not something abstract or imaginary: personal beliefs can actually be empirically investigated.

The fact that an individual's beliefs are accessible for empirical investigation should be of particular interest to the (forensic) cognitive scientists and psychologists involved in this discussion.[5] In this context, we find it relevant to note and worthy a moment of reflection that statisticians, too, have made this observation. Lindley, for example, wrote:

> "My view is that the most important statistical research topic as we enter the new millennium is the development of sensible methods of probability assessment. This will require co-operation with numerate experimental psychologists and much experimental work. A colleague put it neatly, though with some exaggeration: 'There are no problems left in statistics except the assessment of probability'. It is curious that the typical expert in probability knows nothing about, and has no interest in, assessment." [26, at p. 318]

2.5. Why and how any probability statement can always be reconstructed as a subjective probability

All statistical conclusions are obviously conditional on stated assumptions, and this is valid also for frequentist procedures, that are not dispensed from subjective choices such as the hypotheses of interest, the statistical model and so on. In his latest contribution to this article collection, Morrison [29] advocates a procedure by which all subjective elements – concerning what he calls 'pre-empirical matters' (e.g., choice of hypotheses, selection of data) – are separated from the purely technical procedure designed to work out probabilities (or, probability densities) in the numerator and denominator of the likelihood ratio. The result is an abstract procedure whose "(...) output (...) is reported as the strength of evidence statement" [29]. By adding that "(...) such procedures do not involve additional subjective judgement" [29] the stated goal is to work towards 'maximising objectivity'. Although this procedure is transparent with respect to what it is doing, the determination of the probabilities for the evaluation of the strength of evidence in this way raises the following interesting question:

> What do such probabilities mean?[6]

Morrison replies only indirectly to this question by stating that the "(...) proposed solution does not require the forensic practitioner to adopt a subjectivist concept of probability" [29].

We caution against Morrison's statement because it suggests that one can choose the kind of probability that one is using or, stated otherwise, actively decide that one's probability has nothing to do with subjectivism. Our position is

[4] We have never claimed that 'only' subjective probability is legitimate. We have only noted that all other interpretations are less comprehensive and encounter more limitations, mainly regarding applicability [8]. See also Section 2.6 for further discussion on this point.

[5] See [37] for examples on the study of expert probability assignment.

[6] For an insightful discussion of this question, see also [4].

to say that the contrary is actually true. As noted in our last contribution [12], the scientist will need to *decide* on the probabilities, if any, that he will report and retain for the determination of the strength of evidence (i.e., probabilities that make up a likelihood ratio). The reported probabilities may be derived by any legitimate procedure, including those advocated by Morrison. But when it comes to discuss the meaning of the probability or likelihood ratio that is reported, the subjectivist position becomes inevitable.

For the single number that the scientist decides to report – representing the probability assigned for an uncertain event of interest[7] – it is always possible to explain this number with respect to a standard for probability, such as the proportion of balls in an urn that are of a certain colour [e.g., 27]. For example, if the scientist says that their probability for the evidence, given the prosecution's proposition and the relevant information available, is 0.01, we can reconstruct this probability – conceptually – as the scientist's personal point of indifference with respect to the event of successfully drawing[8] a red ball from an urn composed of 1 red ball and 99 white balls: the two events, the evidence and the selection of a red ball, are considered equally uncertain. Again, as noted in Section 2.4, there is no suggestion in our argument that probability assignments in forensic science should be turned into a gambling exercise. We are only pointing out what, conceptually, a given single reported probability means; there would be no hope of making sense out of the complexities and case-specific circumstances of real world forensic cases if we were not able to tell, in principle, how the nature of a reported probability is (to be) understood.

2.6. Why the subjective interpretation of probability is a justifiable premise

In Section 2.5 we have explained why the subjective interpretation of probability is essential when it comes to explaining what a particular expert probability assertion means. The interpretation of probability as an individual's personal belief receives, however, ongoing challenge. Morrison notes:

> "Personally, I find the arguments of Biedermann et al. to be unconvincing because those arguments are based on a premise which a priori I believe to be false, and they have presented no evidence which has convinced me otherwise. The premise is that only a subjectivist concept of probability is legitimate. Under this premise, probability is a state of mind, not a state of nature." [29]

Our reply to this is, first, that the choice of the subjectivist interpretation of probability does not reduce to a mere question of premise. In particular, our premise is *not* 'only a subjectivist concept of probability is legitimate'. In fact, the contrary is true: our starting point is to say that one is entirely free in the choice of one's interpretation of probability, *but* one ought to be able to cope with the implications of the interpretation that one has chosen. Thus, when we say that we choose the subjectivist interpretation of probability, we do so because all other interpretations we know of run into difficulties of various kinds that we want to avoid.

Within the limited scope available in this paper, consider briefly the following two alternative interpretations (see [8] for more detailed discussion): the frequentist interpretation and the classical definition following the works of Pierre-Simon Laplace. In the general context of the law, it has long been understood that the frequentist interpretation of probability does not always work. Almost half a century ago, Kaplan noted: "Given a typical contested trial, for instance, it is meaningless to speak of the probability of the defendant's guilt in terms of the number of times he would be guilty in an infinite number of exactly similar cases (...)" [22, at p. 1066] Similarly, Lindley has noted: "There is nothing wrong with the frequency interpretation, or chance. It has not been used in this treatment because it is often useless. What is the chance that the defendant is guilty?" [25, at p. 48] Frequentist probabilities, thus, are not meaningful for consideration of the probability of legal hypotheses. However, there can be a role for their use in the determination of likelihood ratios. For example, frequentist ideas may be used when prior parameters in the Bayesian calculations involved in the determination of a likelihood ratio are estimated from appropriate databases. Frequentist ideas may then be integrated into a subjective judgement, as noted towards the end of Section 2.3, especially with reference to the quote of De Finetti [14, at p. 334]. In this perspective, frequentist ideas may help in the assignation

[7] We note that this also holds for conditional probabilities as used in a likelihood ratio.

[8] We insist on the point that no repetition (and hence frequentist idea) is involved in this concept. The ball is to be drawn once and once only, with the outcome (and its probability) depending only on the proportion.

of probabilities, but not uniquely so – only as one consideration among others. Note, however, that care is required because many of the controversial issues discussed through this Special Issue, and the wider forensic practice [34], are actually problems that derive from adherence to frequentist concepts.

The second candidate interpretation, the classical definition, is based on the assumption of 'equally probable cases', a concept for which no independent definition is given. This makes this interpretation circular, which is why it is not pursued further.

Our second line of reply to Morrison's skepticism is that, more generally, the judgement of probability interpretations is not a straightforward task because it raises the question of how to specify criteria for the judgment of the adequacy of an interpretation of probability. The criterion of applicability is one example and we have briefly mentioned above why the frequentist interpretation does not always meet this requirement. But the question of how to make a transition from probability in its uninterpreted form to subjective probability touches on the more fundamental issue of how to view mathematics. One relevant perspective in our context of discussion is the so-called intuitionist-constructive attitude. It says that mathematics is a formal language with every concept, starting with the most basic concepts, having a precise meaning. In this perspective, subjective probability is not a function on a space of sets, as in Kolmogorov's axioms, but rather a price assessed for specific economic transactions depending on the truth or otherwise of the event assessed [e.g., 23]. This brings us back to the point that we have made earlier in Section 2.5: subjective probability satisfies an operational definition. It is operational in the sense that it allows us to render explicit the personal views of a person, which are otherwise not directly observable, contemplating aspects of the real world.

In summary, we conclude that a commitment to the subjective interpretation of probability is not a deliberate choice. It is the result of analyzing different concepts of probability and then finding that *constrained* (or 'justified' as we call it in this paper) subjective probability represents advantages over other interpretations that may run into applicability problems. Avoiding such applicability problems is important because they are one reason why probabilism in forensic science and the law has received, and continues to receive, much criticism. Even strong challengers of probability concepts among legal scholars consider that subjective probability is the least problematic perspective: "None of the conceptualizations of probability except probability as subjective degrees of belief can function at trial." [2, at p.104]

2.7. Why existing precepts for evidential assessment are sufficient for keeping the entire field of forensic science engaged

Over the past few decades, several parts of the scientific and legal literature converged in their opinions that scientific evidence should be evaluated in a balanced, transparent, value-adding, robust and logical way [e.g., 21].[9] These are general criteria, formulated independently from particular forms of scientific evidence and specific procedural prescriptions that may vary across different legal systems. The idea behind these criteria is to focus on how scientists can be supported in their sound reasoning about scientific evidence, but recognising that the question of how evidence should actually be *presented* at trial is a different issue on its own (see, e.g., Redmayne [31] on what he calls the 'presentation problem'). It has been gradually recognised, and agreed, that the scientist can work towards meeting the above criteria, especially logic and balance, if his thinking is directed towards answering the questions entailed by the numerator and denominator of the likelihood ratio: what is the probability of the scientist's results if one proposition for the event of interest is true rather than if an alternative proposition is true?

In his latest discussion paper, Morrison [29] proposes his own criteria for evidential assessment:

> "In considering what would be the best approach for evaluating strength of evidence, I suggest that the *desiderata* be (1) to maximise empirically demonstrable performance; (2) to maximise objectivity in the sense of maximising transparency and replicability, and minimising the potential for cognitive bias; and (3) to constrain and make overt the forensic practitioner's subjective-judgement based decisions so that the appropriateness of those decisions can be debated before the judge in an admissibility hearing and/or before the trier of fact at trial." [29]

[9] These criteria are also mentioned in Guidance Note 1 of the ENFSI Guideline [16], and previously in documents issued by the Assocation of Forensic Science Providers [3] and the Royal Statistical Society's Working Group on Statistics and the Law [1].

Implicit in this proposal is the assignment of a likelihood ratio, a purpose that is already covered by the above mentioned existing criteria, though Morrison's desiderata may be seen as providing a more explicit statement not only of the goals, but also of the way in which they ought to be realised, and of the technical properties to which the resulting expressions of probative value should conform. Among these properties is the notion of 'empirical demonstrable performance'. On this notion, we make two comments.

First, the feature of demonstrability can be seen as being part of the existing criteria, mainly robustness. In its broadest sense, robustness is a criterion that challenges a scientist's ability to explain the grounds for his opinion, together with his degree of understanding of the particular evidence type [21], asking him to convey a clear idea of the reasons that he believes entitle him to arrive at the stated conclusions. Jackson has noted:

> "(...) the scientist is directed to challenge the available data and to use it in an expert way. If there is limited data, this should be reflected in the evaluation of the strength of the evidence. The approach not only helps guard against overstating the value of evidence but also assists obtaining maximum value from the evidence." [21, p. 85]

Second, the goal of empirical demonstrability is laudable and may work well in areas of forensic science, especially Morrison's speciality of comparative analyses of speech, where ample potential exists for conducting case-tailored experiments (and the production of scores) under controlled conditions. However, this does not correspond to the starting point of many areas of forensic science. A practical example helps to illustrate this point. Consider the case *R* v *George* [15]. The main scientific finding in this case was a single gunshot residue particle from the internal right pocket of the coat of Mr George, found hanging on the kitchen door of his flat, one year after killing (i.e., single shot to the head of the victim). The following text is a section of one of the scientific reports, given as a direct quote in the judgment (note that FDR is short for firearm discharge residue; bold underline as in original):

> "Conclusion
>
> The significance of the FDR findings in this case can be put into context by considering two alternative propositions:
>
> Mr George is the man who shot Ms Dando
> Mr George had nothing to do with the incident.
>
> In our opinion the probability of finding a single particle of discharge residue in Mr George's coat pocket would have been the same, regardless of which of the above propositions was true.
>
> **The FDR evidence is thus inconclusive. In our opinion it provides no assistance to anyone asked to judge which proposition is true**." [15, par. 23]

The scientists gave here *their* probabilities for the finding given each of the two propositions: they said they consider the two probabilities to be the same, meaning that their likelihood ratio was 1. Clearly, there is no such thing here as an empirical demonstration of their opinion, because it would not feasible. For obvious time and monetary constraints, one cannot reasonably set up controlled experiments of shootings and then wait, in each experiment, one year – because this is an essential feature of the circumstances of the case at hand – to see how much FDR would be found in the pocket. What the experts do is ask the right questions: the probability of the finding, given each of the two propositions of interest. The question is put to experts because they are considered more knowledgable about the particular trace type – here, FDR formation, transfer, persistence and background (on clothing) – than laypersons and the court. There may be debate about whether scientists should be allowed to bring this specific knowledge of trace type into the process. But two points need to be kept in mind. First, if experts' accounts are not admitted, then there is no better person left to testify on the FDR finding. Second, if the experts' accounts are admitted, the relevant questions we want them to consider are still those mentioned above, because they ensure balance, transparency and logic. Conceptually, though experts' specialised knowledge is incomplete, at least to some degree, on practical matters this incompleteness is made formally – and hence scientifically – precise by probability. Note however, that only the subjective interpretation of probability works in this case; the frequentist interpretation would not work. Moreover, the paucity of the data will remain the same, whatever perspective is taken.

As can be seen, Morrison's desiderata would encounter applicability problems in the above example, and it would be detrimental to the field of FDR analysis if this impasse would be used as an argument against the field's pursuit of the general logical framework of evaluative precepts that we have mentioned at the beginning of this section. This risk is real, especially in the area of FDR analysis (see [17] for a discussion). If our intention is to keep the whole field of forensic science on track towards the implementation of the principles of logical evaluative reasoning – which is dependent on asking the right questions in the first place – the general and established precepts for evidential assessment [1, 3, 16, 21] pose a feasible and realistic level of challenge.

With our above two comments we do not seek to advise against empiricism. Investigating the properties of likelihood ratios is insightful and valuable, but it may be done in many ways (see also [36] in this Special Issue) and this can be considered, and should be encouraged, as being part of the established criterion of robustness. We also do not intend to give a pass to 'bad science', we are only arguing that there are branches of forensic science, and cases with peculiar circumstances (especially not replicable events), where the robustness and the foundations of scientists' evaluations require other forms of probing than what Morrison's desiderata suggest, though posing possibly higher challenges to both scientists and cross-examiners.

2.8. Why it is important to understand the personalistic position as part of the concept of probabilistic modelling

For the most part in this paper we have focused on the nature and inevitability of probability to be understood as a personal assessment. In many instances, probabilities are assigned through a formal model, involving one or more parameters for which, too, the scientist needs to specify probability distributions. So again, we see that the scientist has an active role in the process of probabilistic modelling. In our previous papers in this Special Issue we have not addressed, for example, the particular aspect of elicitating prior distributions for parameters. Morrison [29] approaches this topic in his last contribution. Clearly, any statistical procedure is characterized by subjective elements, such as the choice of hypotheses, model assumptions, and also the choice of prior distributions for model parameters. It is well understood that, given available data, hypotheses and model assumptions, a change in a prior distribution may lead to appreciable changes in the output, which may be the value of a likelihood ratio or the distribution of a parameter of interest. This is an important topic known as sensitivity analysis, however, it was not our main focus of attention. Morrison's latest proposal goes into the direction of 'objectifying' a Bayesian solution with the aim of avoiding informative prior distributions as they may be felt too arbitrary and hence not useful (or justifiable) in a legal setting. This touches on yet another topic we did not pursue, the aversion to priors. This topic is dealt with extensively elsewhere in existing literature (see, for example, Bernardo [6] and related literature for a discussion about reference analysis). We underline, however, that subjectivism is unavoidable once we embrace the Bayesian paradigm that is characterized by an interpretation of probability as a rational, conditional measure of uncertainty. Each element in a Bayesian analysis, including prior information, has a well-defined role and trying to eliminate any element would mean to misconceive what the theory is actually trying to achieve. As noted by Howson [20]: "(...) to deliberately suppress it [prior information] in a theory which gives it an explicit role is perverse (....)" [p. 56].

3. Conclusions

We have come to a strange point in this discussion. One the one hand, all discussants accept that probability is the appropriate scientific way to quantify incomplete knowledge or uncertainty. On the other hand, there are fundamental disagreements about how a forensic expert's assertion of probability may be understood. These disagreements do not help to strengthen the perception of the trustworthiness of forensic science. They do not help either to further the roles of science [19] and probability [8] in forensic practice. Yet we see room for a reconciling perspective: the perspective is that all discussants pursue the idea that we should make the best use of relevant data. Differences arise only over the question of how exactly this ought to be done:

- Proponents favouring a strong empiricist or frequentist perspective use data exclusively to define probability – with the consequence of not being able to give a probability in situations that cannot meaningfully be reconstructed in an experimental way. In short this position says, no probability – or, 'I don't know what the probability is' – without data.

- This is different for supporters of subjective probabilism whose probability can always be given (including for single, not repeatable events), for it would not make sense for them to say that they do not know their own mind, nor that they do not know what their probability is. Subjective probability is as much informed by data as other types of probability, but it is not *defined* by that data. The data only informs, or conditions, an expert's probability.

Considering the challenges posed on a practical and operational account, the issue of warrant is particularly noteworthy [28]. As we have argued, however, warrants for probability assertions do not concern only subjective probability, but all types of probabilities conveyed by scientists. Thus, recipients of expert information should always probe the foundations of any probabilities asserted by scientists.

A further conclusion that may be drawn is the need to be precise regarding the terminology and definition of concepts. For example, many standard criticisms levelled against subjectivism invoke the notion of guesswork: given the fact that subjective probabilities purport to represent an individual's personal beliefs which, by definition, can take any value, it is argued that subjective probabilities run the risk of being aleatory, fanciful, speculative or otherwise unsound. However, the discussion is not about so-called unconstrained subjective probabilities; that would indeed be discomforting. As noted above, the concept we have in mind is of constrained subjective probability. Probabilities of this type, conditioned on task-relevant data, are also required by current evaluative guidelines [16].

A generally skeptical position is taken by Morrison [29] who contends that "[i]nsisting on a subjectivist concept of probability is counterproductive" and would go against a "logically correct and scientifically valid evaluation of forensic evidence". Our reply (Section 2.4) to this is that, first, subjective probabilities are, of course, logical (i.e., in agreement with the rules of probability calculus) and scientific (i.e., measurable). Second, subjective probability is not a question of insistence, but of inevitability (Section 2.5), and it would be unscientific actively to ignore this reality.

While no-one is able to tell scientists what their subjective probabilities ought to be, clarification can be given as to the consequences of their choices. Ultimately, the probability that a scientist will retain in his reporting – in whatever way it was worked out and its nature is understood – can be seen as a decision [9, 12], highlighting the idea that the reported probability represents the answer *judged* most appropriate – conditioned on all task-relevant information, domain knowledge and background data – that the scientist can give in reply to the uncertainty to be assessed. Understanding expert probability reporting as a decision to be made by the scientist has the additional advantage of encouraging scientists to make up their minds seriously, and assume responsibility for the assignments they will convey. As in probability theory, we cannot require all discussants to share the same probability in actual tasks requiring probability assignments, though it appears fair to require all discussants to have reasonable (i.e., justified) probabilities [18]. Broad agreement on this is demonstrated in this Special Issue by the fact that there is ample exchange on probability assignment, which also demonstrates that discussants take this task seriously and actually want their probabilities to be well supported.

Acknowledgments

To be added.

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